Empirical Diffusion Coefficients for Natural-Uranium CANDU Lattices

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Outline

- Introduction
- Objective
- Approach
- Calculations
- Results
- Conclusion
- Future Investigations

Full Core Calculations

- Successive approximations
  1. many-group heterogeneous transport (reference)
  2. two-group node-homogenized transport
  3. two-group node-homogenized diffusion
- Observation: The difference between step 1 and step 2 results is (much) smaller than the difference between step 2 and step 3 results.
- **Initial Objective:** Adjust diffusion coefficient so that two-group node-homogenized diffusion results closely match two-group node-homogenized transport results for CANDU lattices.

Broader Objective

- Develop equivalence between node-homogenized two-group transport model and two-group ***diffusion-like*** model**,** such that the node-integrated reaction rates calculated using the two models are the same.

Angle-Integrated Neutron Balance Equation

- Transport

$$\nabla\cdot\vec{J}_g^{tr}(\vec{r})+\Sigma_{rg}(\vec{r})\Phi_g^{tr}(\vec{r})-\sum_{g'\neq g}\Sigma_{g'\to g}(\vec{r})\Phi_{g'}^{tr}(\vec{r})=\frac{1}{k_{eff}^{tr}}\chi_g\sum_{g'}\nu\Sigma_{fg'}(\vec{r})\Phi_{g'}^{tr}(\vec{r})$$

- Diffusion

$$-\nabla\cdot\left[D_g(\vec{r})\nabla\Phi_g(\vec{r})\right]+\Sigma_{rg}(\vec{r})\Phi_g(\vec{r})-\sum_{g'\neq g}\Sigma_{g'\to g}(\vec{r})\Phi_{g'}(\vec{r})=\frac{1}{k_{eff}}\chi_g\sum_{g'}\nu\Sigma_{fg'}(\vec{r})\Phi_{g'}(\vec{r})$$

Comments

- Adjusting diffusion coefficients to achieve direct equivalence between transport and diffusion in 3D requires directional diffusion coefficients.
- Equivalence is much easier to achieve when using a simplified diffusion equation:

$$-D_g(\vec{r})\nabla^2\Phi_g(\vec{r})+\Sigma_{rg}(\vec{r})\Phi_g(\vec{r})-\sum_{g'\neq g}\Sigma_{g'\to g}(\vec{r})\Phi_{g'}(\vec{r})=\frac{1}{k_{eff}}\chi_g\sum_{g'}\nu\Sigma_{fg'}(\vec{r})\Phi_{g'}(\vec{r})$$

New Problem

- Find equivalence between simplified diffusion and transport in two energy groups for CANDU lattices.

$$\nabla \cdot \vec{J}_g^{tr}(\vec{r}) + \Sigma_{rg}(\vec{r})\Phi_g^{tr}(\vec{r}) - \sum_{g' \neq g} \Sigma_{g' \to g}(\vec{r})\Phi_{g'}^{tr}(\vec{r}) = \frac{1}{k_{eff}^{tr}} \chi_g \sum_{g'} \nu\Sigma_{fg'}(\vec{r})\Phi_{g'}^{tr}(\vec{r})$$

$$\Updownarrow$$

$$-D_g(\vec{r})\nabla^2\Phi_g(\vec{r}) + \Sigma_{rg}(\vec{r})\Phi_g(\vec{r}) - \sum_{g' \neq g} \Sigma_{g' \to g}(\vec{r})\Phi_{g'}(\vec{r}) = \frac{1}{k_{eff}} \chi_g \sum_{g'} \nu\Sigma_{fg'}(\vec{r})\Phi_{g'}(\vec{r})$$

Equivalence

- Transport Equation

$$\nabla \cdot \vec{J}_g^{tr}(\vec{r}) + \Sigma_{rg}(\vec{r})\Phi_g^{tr}(\vec{r}) - \sum_{g' \neq g} \Sigma_{g' \to g}(\vec{r})\Phi_{g'}^{tr}(\vec{r}) = \frac{1}{k_{eff}^{tr}} \chi_g \sum_{g'} \nu\Sigma_{fg'}(\vec{r})\Phi_{g'}^{tr}(\vec{r})$$

- Define: $D_g(\vec{r}) = -\frac{\nabla \cdot \vec{J}_g^{tr}(\vec{r})}{\nabla^2 \Phi_g^{tr}(\vec{r})} \Rightarrow -D_g(\vec{r})\nabla^2\Phi_g^{tr}(\vec{r}) = \nabla \cdot \vec{J}_g^{tr}(\vec{r})$

- Transport flux satisfies simplified-diffusion eq.

$$-D_g(\vec{r})\nabla^2\Phi_g^{tr}(\vec{r}) + \Sigma_{rg}(\vec{r})\Phi_g^{tr}(\vec{r}) - \sum_{g' \neq g} \Sigma_{g' \to g}(\vec{r})\Phi_{g'}^{tr}(\vec{r}) = \frac{1}{k_{eff}} \chi_g \sum_{g'} \nu\Sigma_{fg'}(\vec{r})\Phi_{g'}^{tr}(\vec{r})$$

Comments

- If the transport flux is known, a simplified diffusion problem can be set up whose solution produces the same integral flux as the transport problem.
- Possible difficulties setting up the simplified diffusion problem:
  - Transport flux not known beforehand
  - Transport flux may be known in a form that is not amenable to calculation of Laplacian (e.g. irregular-region CP)

CANDU-Problem-Specific Features

- Equivalence is sought between two ***node-homogenized*** models.
- For homogenized nodes a simple Cartesian mesh can be used, which allows easy computation of Laplacian.
- Homogenized diffusion coefficients in CANDU lattices vary little with local conditions and hence could be approximated to be constant in space.

Simplification

- Assumption
  - Diffusion coefficients for the simplified diffusion equation are almost constant throughout the system and have very similar values regardless of the system configuration.
- Consequences
  - Using constant (average) diffusion coefficients throughout a system yields almost the same results as using the position-dependent ones.
  - Average simplified-diffusion coefficients (one value for each group) can be calculated for a simple configuration and then applied to any other configuration.

Determining the Empirical Diffusion Coefficients

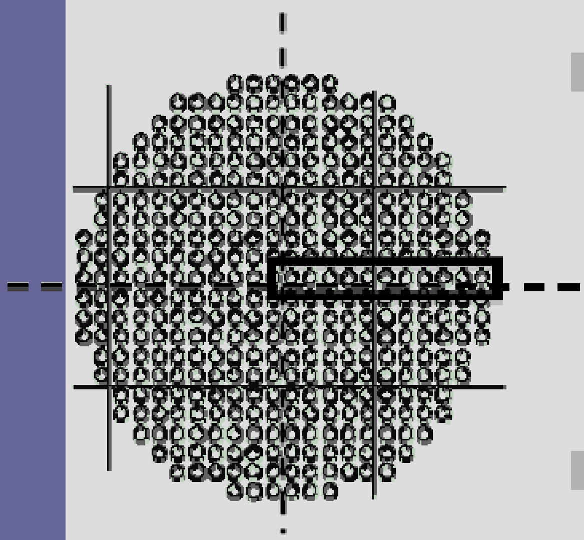
- For a simple configuration, calculate:

$$\overline{D}_g = \frac{\int w_g(\vec{r}) \nabla \cdot \vec{J}_g^{tr}(\vec{r}) d^3 r}{\int w_g(\vec{r}) \nabla^2 \Phi_g^{tr}(\vec{r}) d^3 r} \cong D_g(\vec{r}) = \frac{\nabla \cdot \vec{J}_g^{tr}(\vec{r})}{\nabla^2 \Phi_g^{tr}(\vec{r})}$$

- If currents are not directly available, use:

$$\overline{D}_g = \frac{\int w_g(\vec{r}) \left[\Sigma_{rg}(\vec{r}) \Phi_g^{tr}(\vec{r}) - \sum_{g' \neq} \Sigma_{g' \to g}(\vec{r}) \Phi_{g'}^{tr}(\vec{r}) \right] d^3 r - \frac{1}{k_{eff}^{tr}} \int w_g(\vec{r}) \chi_g \sum_{g'} \nu \Sigma_{fg'}(\vec{r}) \Phi_{g'}^{tr}(\vec{r}) d^3 r}{\int w_g(\vec{r}) \nabla^2 \Phi_g^{tr}(\vec{r}) d^3 r}$$

Configurations (1-D)



- For calculating $\overline{D}_g$
  - 11 identical homogeneous-nodes (corresponding to mid-burnup fuel)

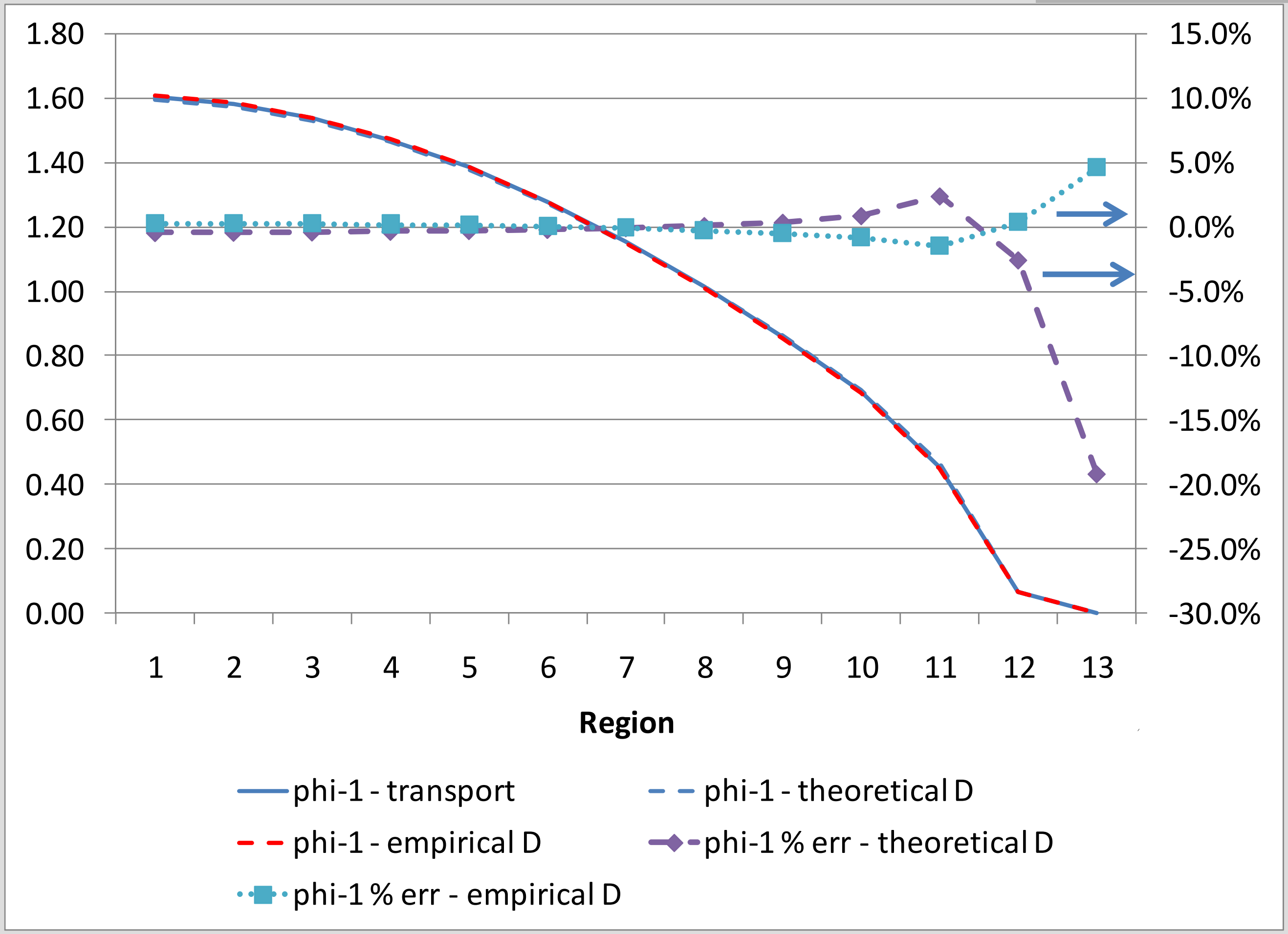
REFL | F | F | F | ---------- | F | VAC

- For testing
  - 13 homogeneous nodes (11 fuel + 2 reflector)

REFL | F | F | F | ---------- | F | R | R | VAC

REFL | F | F | F | ---------- | F | R | R | VAC

All Mid-Burnup Fuel + Reflector Fast Flux



Results

UOIT CHALLENGE INNOVATE CONNECT

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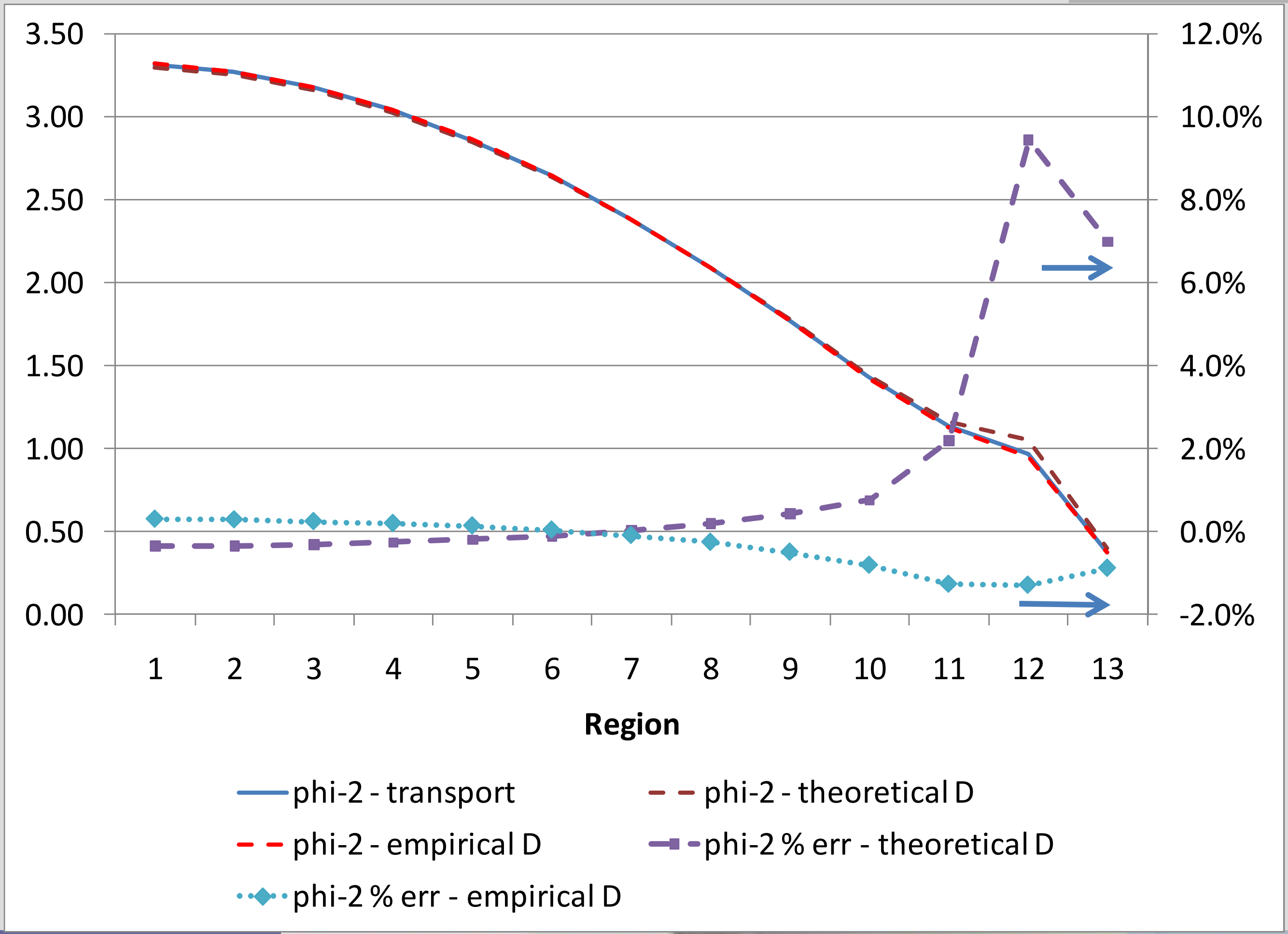
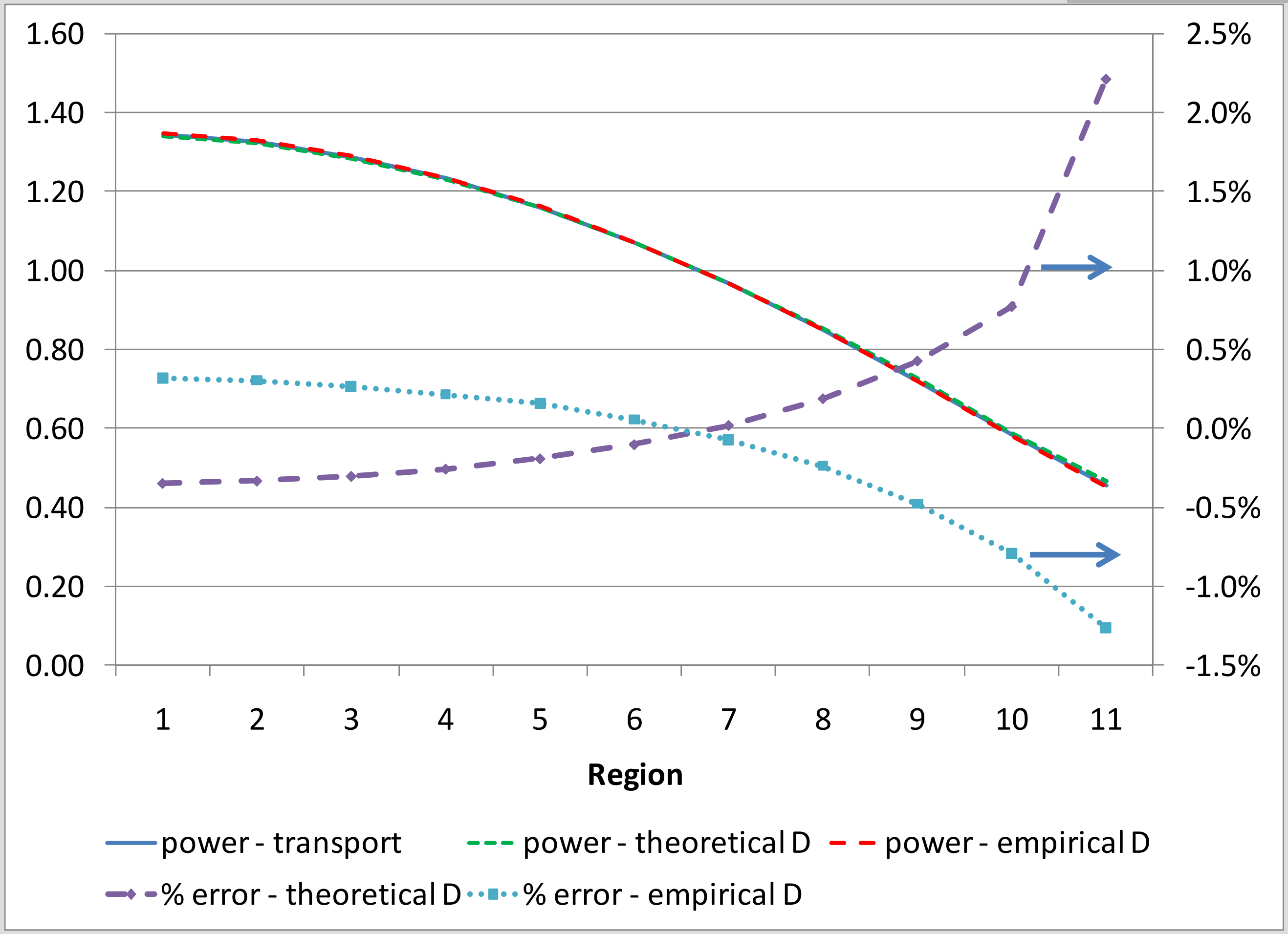
All Mid-Burnup Fuel + Reflector Thermal Flux



Results

All Mid-Burnup Fuel + Reflector Power

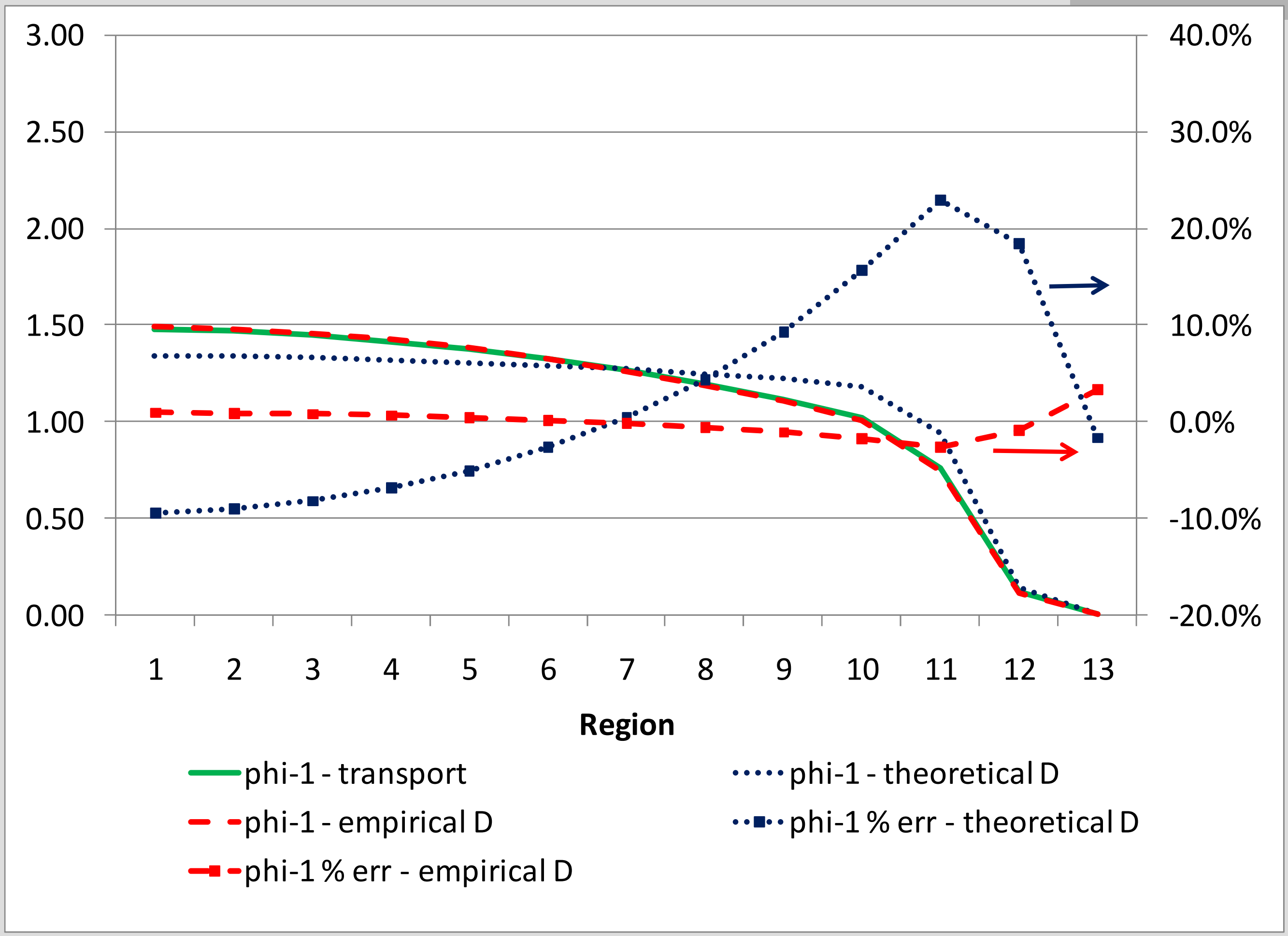


Results

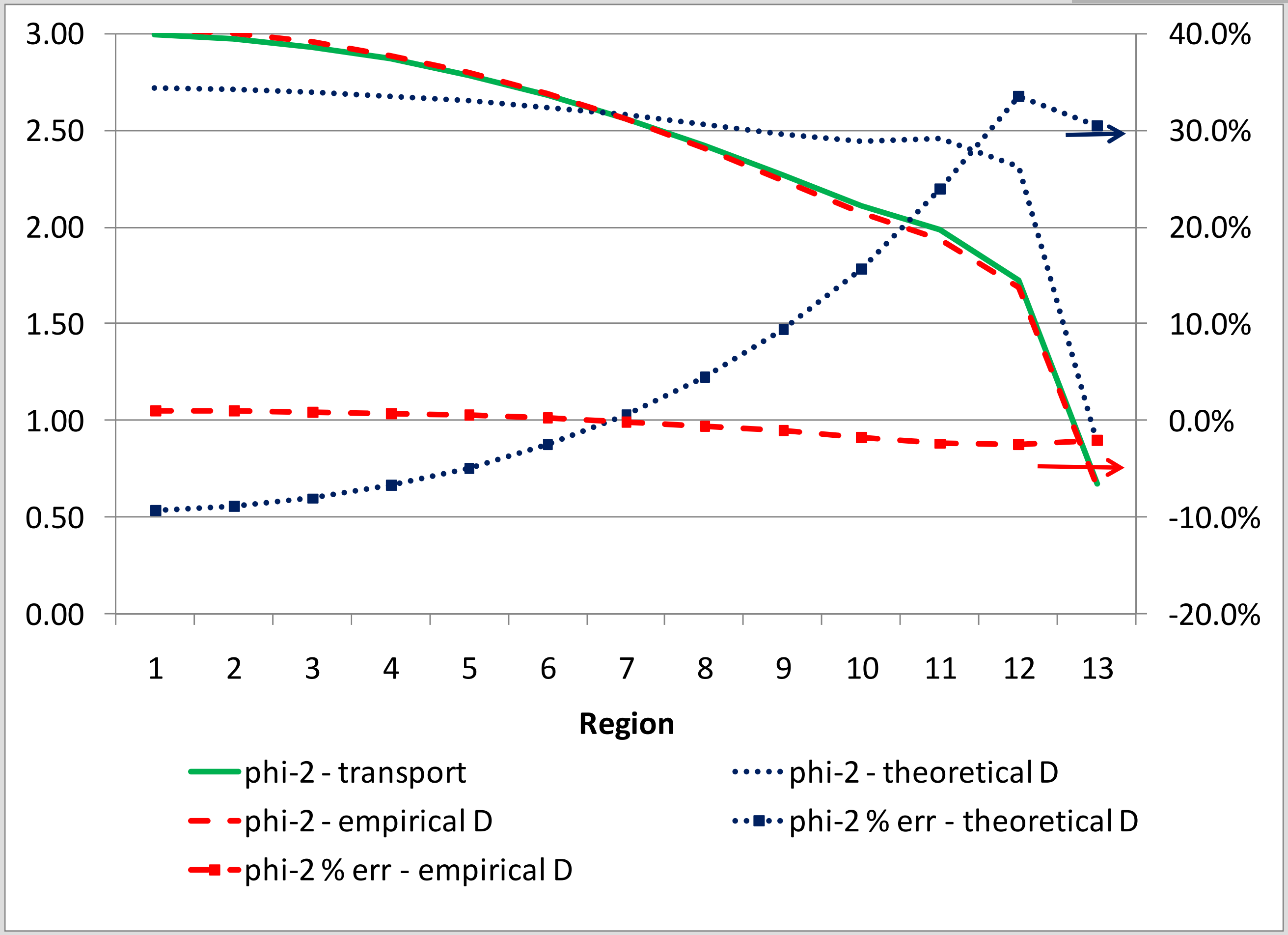
All Mid-Burnup Fuel + Reflector Error Synopsis

		D calculation method	
		theoretical	empirical
k_{eff} error	(mk)	1.6	-0.1
Φ_1 error	max	19.2%	4.6%
	RMS	5.4%	1.4%
Φ_2 error	max	9.4%	1.3%
	RMS	3.3%	0.6%
power error	max	2.2%	1.3%
	RMS	0.7%	0.5%

Disch. Fuel + Fresh Fuel + Refl. Fast Flux



Disch. Fuel + Fresh Fuel + Refl. Thermal Flux



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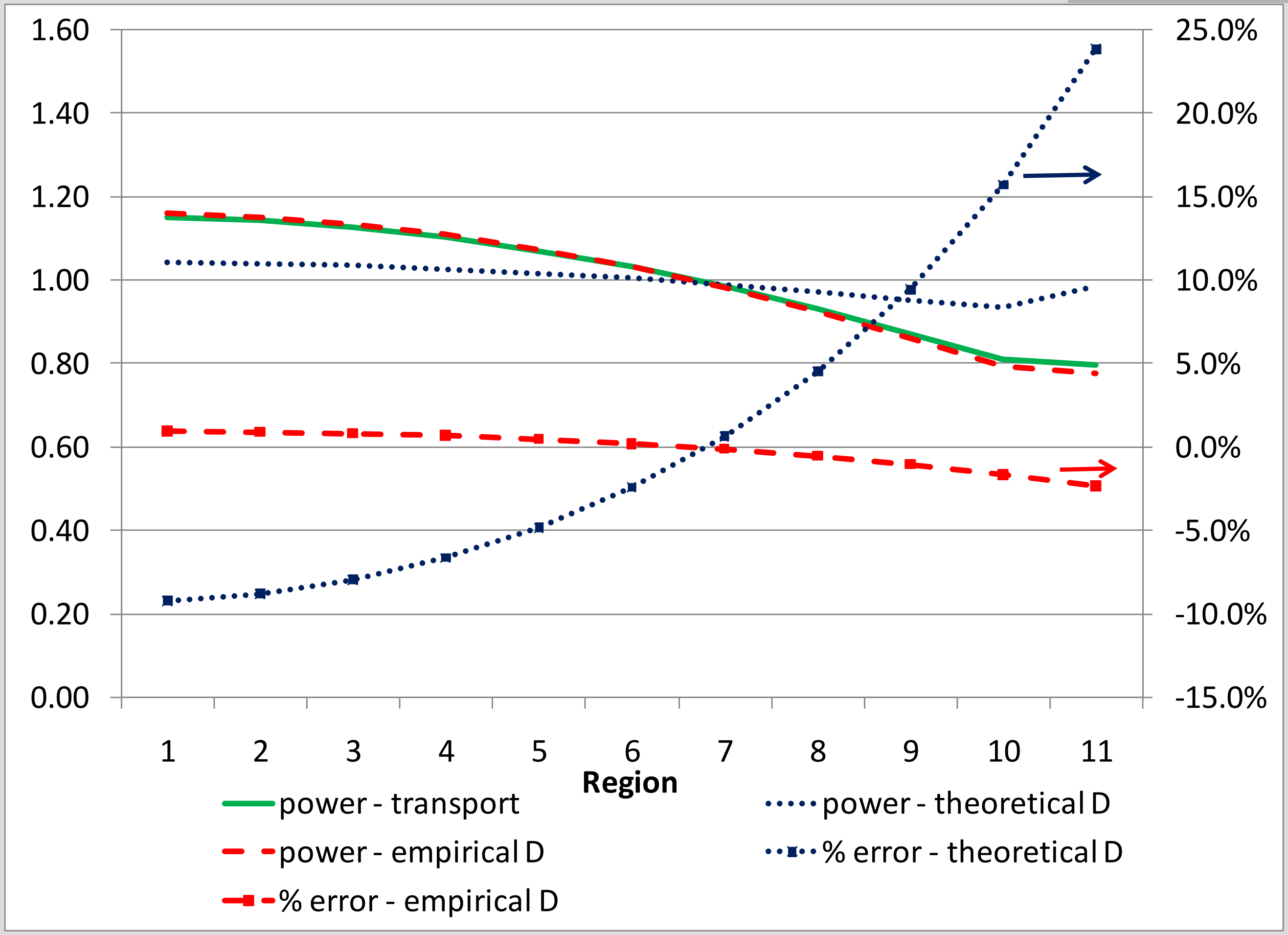
Disch. Fuel + Fresh Fuel + Refl. Power



Results

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Disch. Fuel + Fresh Fuel + Refl. Error Synopsis

		D calculation method	
		theoretical	empirical
k_{eff} error	(mk)	2.5	-0.2
Φ_1 error	max	22.9%	3.4%
	RMS	10.8%	1.4%
Φ_2 error	max	33.5%	2.4%
	RMS	15.9%	1.3%
power error	max	23.8%	2.4%
	RMS	10.5%	1.1%

Comments

- Empirical diffusion coefficients allow the simplified diffusion results to match closely transport results.
- Because the diffusion coefficients are assumed constant in space, existing codes do not need to be modified to implement the simplified diffusion equation.

Conclusion

- Using empirical diffusion coefficients for CANDU-lattice node-homogenized models yields substantial gains in accuracy for simple, one-dimensional, configurations.

Future Investigations

- Confirm improvement in accuracy occurs for two- and three-dimensional models.
- Develop an interpretation of the difference between theoretical and empirical diffusion coefficients.

Questions